## Model Geometry

## Calculating Angles Associated with Skewed Columns Using Vector Notation and Direction Cosines



This example uses vector notation to determine the angles associated with a skewed column.


Actual Length (in) 20.445
Eff. Length $z z$ (in) 20.445 UD
Eff. Length yy (in) 20.445 UD
Offset $z$ (in) : 0
Offset y (in) : 0 Roll Angle 0 Degree

Let's consider member number 3.
From the "Member Properties" dialog we see that this member starts at joint number 1 and ends at joint number 8. The actual length of the member is 20.445 inches.


From the "Joint Properties" dialog we see that joint number 8 has the following coordinates:

X=9
$Y=2$
Z=18
For the calculations let's call this location node 1 and use the following notation:
$X_{1}=9 \quad Y_{1}=2 \quad Z_{1}=18$


From the "Joint Properties" dialog we see that joint number 8 has the following coordinates:

X=12
$\mathrm{Y}=22$
$Z=15$
$X_{2}=12 \quad Y_{2}=22 \quad Z_{2}=15$

Using vector notation to represent the column we get the following:

## $\mathbf{C = d X i}+d Y \mathbf{j}+d Z \mathbf{k}$

where the bold " $C$ " denotes a vector with a change in the global " $X$ " direction directed along a unit vector $\mathbf{i}$ pointing the global $X$ direction.
$\mathrm{dX}=\mathrm{X}_{2}-\mathrm{X}_{1}=12-9=3$ (change in the $\mathbf{i}$ direction)
$d Y=Y_{2}-Y_{1}=22-2=20$
$d X=X_{2}-X_{1}=15-18=-3$
$\mathbf{C}=3 \mathbf{i}+20 \mathbf{j}-3 \mathbf{k}$

The magnitude of vector $\mathbf{C}$ (denoted as $\mathbf{C}$ ) is:
$\mathrm{C}=\operatorname{sqrt}\left((\mathrm{dX})^{2}+(\mathrm{dY})^{2}+(\mathrm{dZ})^{2}\right)$
$C=\operatorname{sqrt}\left((3)^{2}+(20)^{2}+(-3)^{2}\right)$
C=sqrt(418)
$C=20.445$ (as it should)

Now convert $\mathbf{C}$ to a unit vector directed along the same axis as column 3:

$$
\mathbf{c}=\mathbf{C} / \mathbf{C}=\mathbf{C} / 20.445=(3 \mathbf{i}+20 \mathbf{j}-3 \mathbf{k}) / 20.445=.14674 \mathbf{i}+.97823 \mathbf{j}-.14674 \mathbf{k}
$$

The coefficients of the unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are the angle cosines.

Cosine ${ }^{-1}(.97823)=11.98^{\circ}$ (this is the angle in degrees from the Y direction to the column. The column is inclined by $90-11.98=78.02$ degrees or about $78^{\circ}$

Cosine $^{-1}(.14674)=81.56^{\circ}$ (this is the angle in degrees from the $X$ direction to the column.
$\operatorname{Cosin}^{-1}(-.14674)=-98.438^{\circ}$ (this is the angle in degrees from the $Z$ direction to the column.

